

Gravitational Instability of a Two Component Rotating Viscous Plasma under the Effect of Arbitrary Radiative Heat-Loss Functions and Electron Inertia

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ABSTRACT

The problem of a self-gravitational instability of infinite homogeneous two component Rotating viscous plasma in the presence of magnetic field, electrical resistivity, the effect of arbitrary radiative heat-loss functions and finite electron inertia is investigated. A general dispersion relation is obtained using the normal mode analysis with the help of relevant linearized perturbation equation of the problem and a modified Jeans criterion of instability is obtained. The conditions of modified Jeans instabilities and stabilities are discussed in the different cases of our interest. The effect of electron inertia affects only the longitudinal mode of propagation and it has no effect on the transverse mode of propagation. We find that the Jeans condition of gravitational instability is modified due to the presence of thermal conductivity and density dependent heat-loss function. Numerical calculations have been performed to show the effect of various parameters on the growth rate of the gravitational instability.

Keywords: Rotation, Arbitrary radiative heat-loss functions, partially-ionized plasma, thermal conductivity and collision frequency.

1. INTRODUCTION

The Gravitational instability
Problem of an infinite homogeneous

medium was first considered by Jeans¹. Since then several authors have studied this problem, under varying assumption of hydrodynamics and hydromagnetics, and a

comprehensive account of all these investigations has been given by Chandrasekhar² in his monograph on problem of hydrodynamic stability. He has shown that Jean's criterion remains unaffected by the separate or simultaneous presence of a uniform rotation and uniform magnetic field. Pacholczyk and Stodolkiewicz³ have studied the effect of rotation and finite conductivity on the gravitational instability of an interstellar medium like the HI region. It is, therefore, of importance and is the object of the present investigation to study the effects of finite conductivity and frictional effects with neutrals on the gravitational instability of interstellar and interplanetary plasmas. Chonkar and Bhatia⁴ have studied the combined influence of Coriolis force and viscosity on plasma stability in the absence of Hall currents and concluded that viscosity has a stabilizing influence on the system. Partially-ionized plasma represents a state which often exists in the universe. The interaction between the natural and the ionized gas components becomes importance in the cosmic sphere. In above mentioned studies none of the authors has incorporated the radiative effects in their studies.

Along with this, one of the most important phenomenon in many parts of physics is so called thermal radiation or black body radiation, where the distribution of electromagnetic radiation energy is in thermal equilibrium with the substance pervaded by it. Such radiation is of great astrophysical importance, since many cosmic objects are characterized by high temperature of the order of tens of millions degrees or more, including relativistic

temperatures. The phenomenon of thermal instability arising due to heat-loss mechanisms in a dilute plasma has been studied by several authors. The importance of thermal instability in the formation of condensations like galaxies, solar prominences and planetary nebulae from a dilute gas has been analyzed by Field⁵. Dwivedi *et al.*⁶ has analyzed the effect of radiative condensation on Jeans instability. They pointed out that under certain conditions, the radiative cooling can overwhelm the gravitational collapse. Aggrawal and Talwar⁷ have investigated the problem of incipient fragmentation of interstellar matter to form condensation, taking into account thermal and radiative effects incorporated viscosity, electrical conductivity, rotation and uniform magnetic field. They have pointed out that the classical Jeans criterion regarding the size of initial break up is considerably modified due to a heat-loss function in an in viscid and ideally conducting fluid. In the recent ISM observations, it is observed that the radiative heat-loss mechanism plays an important role in the star formation and molecular cloud condensation process in connection with thermal instability. The ISM structure shows that the heat-loss process plays an important role in the condensation of large astrophysical compact objects. Basically this radiative heat-loss function shows the decay of heat in an embedded system with respect to local temperature and density.

Recently, Shaikh and Khan⁸ have discussed the instability of thermally conducting self-gravitating system. Pensia *et al.*⁹ have discussed the problem of self-gravitational instability in the presence of suspended particles. Pensia *et al.*¹⁰ have

studied the thermal instability of self-gravitating, rotating gaseous plasma with generalized ohm's law. The Jeans instability of rotating anisotropic heat-conducting plasma has been discussed by Prajapati *et al.*¹¹. Uberio¹² has investigated the electron inertia effect on the transverse gravitational instability. More recently, prajapati and Chhajlani¹³ have discussed the effect of dust temperature on both jeans instability and radiative condensation instability (RCI) of self-gravitating magnetized dusty plasma. Yang *et al.*¹⁴ have studied the large-scale gravitational instability and star formation in molecular clouds. The gravitational stability of partially ionized astrophysical plasmas embedded in a large-scale magnetic field has been studied by Jacobs and Shukla¹⁵. Inutsuka *et al.*¹⁶ have investigated the propagation of shock waves into a warm neutral medium taking into account radiative terms. Menou *et al.*¹⁷ have shown the importance of radiative effects in the sun's upper radiative zone. Kim and Narayan¹⁸ have discussed the thermal instability in clusters of galaxies with conduction taking on the role of the effect of radiative heat-loss function. Shadmeri and Dib¹⁹ have discussed the thermal instability in magnetized partially ionized plasma with charged dust particles and radiative cooling functions.

In this paper we have investigated the effect of radiative Heat-Loss function, thermal conductivity, finite resistivity, viscosity and electron inertia on the Jeans gravitational instability of homogenous two component rotating plasma.

2. LINEARIZED PERTURBATION EQUATIONS

We consider an infinite homogeneous, self-gravitating plasma including finite electron inertia, arbitrary radiative heat-loss functions, rotation, viscosity, finite electrical resistivity and thermal conductivity, density of ionized component ρ , density of neutral components ρ_d ($\rho \gg \rho_d$). It is assumed that the above medium is permeated with a uniform magnetic field $\vec{H}(0, 0, H)$. The axis of rotations are taken along and perpendicular to the direction of the magnetic field. We assume that the two components of the partially ionized plasma (the ionized fluid and the neutral gas) behave like a continuum fluid and their state velocities are equal.

Thus the linearized perturbation equations governing the motion of hydromagnetic thermally conducting two component plasma rotating with a uniform angular velocity are given by

$$\rho \frac{\partial \vec{v}}{\partial t} = -\vec{\nabla} \delta p + \rho \vec{\nabla} \delta \varphi + \frac{1}{4\pi} (\vec{\nabla} \times \vec{h}) \times \vec{H} + \rho_d \nu_c (\vec{\nabla}_d - \vec{v}) + \rho \nu \nabla^2 \vec{v} + 2\rho (\vec{v} \times \Omega). \quad (1)$$

$$\frac{\partial \vec{v}_d}{\partial t} = -\nu_c (\vec{\nabla}_d - \vec{v}). \quad (2)$$

$$\frac{\partial \delta \rho}{\partial t} = -\rho \vec{\nabla} \cdot \vec{v}. \quad (3)$$

$$\nabla^2 \delta \varphi = -4\pi G \delta \rho. \quad (4)$$

$$\frac{1}{(\gamma - 1)} \frac{\partial \delta \rho}{\partial t} - \frac{\gamma}{(\gamma - 1)} \frac{p}{\rho} \frac{\partial \delta \rho}{\partial t} + \rho (\mathcal{L}_\rho \delta \rho + \mathcal{L}_T \delta T) - \lambda \nabla^2 \delta T = 0. \quad (5)$$

$$\frac{\delta p}{p} = \frac{\delta T}{T} + \frac{\delta \rho}{\rho}. \quad (6)$$

$$\frac{\partial \vec{h}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{H}) + \eta \nabla^2 \vec{h} + \frac{c^2}{\omega_{pe}^2} \frac{\partial}{\partial t} \nabla^2 \vec{h}. \quad (7)$$

$$\vec{\nabla} \cdot \vec{h} = 0. \quad (8)$$

where the parameters $G, v, \lambda, \eta, R, \gamma, \Omega$ ($\Omega_x, 0, \Omega_z$) c, ω_{pe}, N, e , and v_d denote the gravitational constant, kinetic viscosity, thermal conductivity, electrical resistivity, gas constant, ratio of two specific heats, rotational frequency, velocity of light, electron plasma frequency, number density, charge of electron and neutral gas velocity respectively.

The perturbations in fluid velocity, fluid pressure, fluid density, magnetic field, gravitational potential, temperature and the radiative heat-loss function are given as $\vec{v}(v_x, v_y, v_z), \delta p, \delta \rho, h(h_x, h_y, h_z), \delta \phi, \delta T$, and $\delta \mathcal{L}$ respectively. In equation (5), $\mathcal{L}_{\rho, T}$ are the partial derivatives of the density dependent $(\partial \mathcal{L} / \partial \rho)_T$ and temperature dependent $(\partial \mathcal{L} / \partial T)_\rho$ heat-loss functions respectively.

3. DISPERSION RELATION

We assume that all the perturbed quantity vary as

$$\exp \{i(k_x x + k_z z + \omega t)\}. \quad (9)$$

where ω is the frequency of harmonic disturbances, k_x and k_z are wave numbers in x and z direction, respectively, such that $k_x^2 + k_z^2 = k^2$ combining equation (5) and (6), we obtain the expression for δp as

$$\delta p = \left(\frac{\alpha + \sigma c^2}{\sigma + \beta} \right) \delta \rho. \quad (10)$$

where $\sigma = i\omega$, $c = \left(\frac{\gamma p}{\rho} \right)^{1/2}$ is the adiabatic velocity of sound in the medium. The parameter α and β are given by

$$\begin{aligned} \alpha &= (\gamma - 1) \left(\mathcal{L}_T T - \mathcal{L}_\rho \rho + \frac{\lambda k^2 T}{\rho} \right). \\ \beta &= (\gamma - 1) \left(\frac{\mathcal{L}_T T \rho}{p} + \frac{\lambda k^2 T}{p} \right). \end{aligned} \quad (11)$$

Using equation (2) – (11) in equation (1), we obtain the following algebraic equations for the amplitude components

$$\left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 V^2}{A} \right) v_x - 2\Omega_x v_y + \frac{ik_x}{k^2} \Omega_T^2 s = 0. \quad (12)$$

$$2\Omega_z v_x + \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{k_z^2 V^2}{A} \right) v_y - 2\Omega_x v_z = 0. \quad (13)$$

$$2\Omega_x v_y + \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) v_z + \frac{ik_z}{k^2} \Omega_T^2 s = 0. \quad (14)$$

Taking divergence of equation (1) with add of equation (2) – (11), we obtain

$$\left(\frac{ik_x k^2 V^2}{A} \right) v_x - 2i(k_x \Omega_z - k_z \Omega_x) v_y - \left\{ \sigma \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) + \Omega_T^2 \right\} s = 0. \quad (15)$$

where $s = \frac{\delta \rho}{\rho}$ is the condensation of the medium, $V = \frac{H}{(4\pi\rho)^{1/2}}$ is the Alfvén velocity, $c^2 = \gamma c'^2$ where c and c' are the adiabatic and isothermal velocities of sound. Also we have assumed the following substitutions.

$$B = \frac{\rho d}{\rho}, \quad \Omega_T^2 = \left(\frac{\sigma \Omega_j^2 + \Omega_i^2}{\sigma + \beta} \right), \quad \Omega_i^2 = (k^2 \alpha - 4\pi G \rho \beta),$$

$$\Omega_j^2 = (k^2 c^2 - 4\pi G \rho), \quad \Omega_v = \nu k^2, \quad \Omega_m = \eta k^2,$$

$$d = (\sigma + \beta), \quad A = (\sigma f + \Omega_m), \quad f = \left(1 + \frac{c^2 k^2}{\omega_{pe}^2} \right).$$

The nontrivial solution of the determinant of the matrix obtained from equation (12) to (15) with v_x, v_y, v_z, S having various coefficients that should vanish is to give the following dispersion relation

$$\left[- \left\{ \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 V^2}{A} \right) \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{k_z^2 V^2}{A} \right) \left\{ \sigma \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) + \Omega_T^2 \right\} \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) \right\} - \right.$$

$$4 \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 V^2}{A} \right) \left\{ \sigma \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) + \Omega_T^2 \right\} \Omega_x^2 - \frac{4 \Omega_x k_x k_z \Omega_z \Omega_T^2}{k^2} \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 V^2}{A} \right) +$$

$$\frac{4 \Omega_x^2 k_z^2 \Omega_T^2}{k^2} \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 V^2}{A} \right) - 4 \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) \left\{ \sigma \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) + \Omega_T^2 \right\} \Omega_z^2 +$$

$$\frac{4 \Omega_x k_x k_z \Omega_z V^2 \Omega_T^2}{A} + \frac{4 \Omega_z^2 k_x^2 \Omega_T^2}{k^2} \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) - \frac{4 \Omega_x k_x k_z \Omega_z \Omega_T^2}{k^2} \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) + \frac{V^2 k_x^2 \Omega_T^2}{A} \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \right.$$

$$\left. \Omega_v \right) \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{k_z^2 V^2}{A} \right) + \frac{4 V^2 k_x^2 \Omega_x^2 \Omega_T^2}{A} \left. \right] = 0. \quad (16)$$

The dispersion relation (16) shows the combined influence of partially ionized plasma, finite electron inertia, viscosity, rotation, thermal conductivity, electrical resistivity and arbitrary radiative heat-loss functions on the gravitational instability of a homogeneous plasmas. If we ignore the effects of the partially ionized plasma then we contribute Hall parameter and permeability. The present results are also similar to those of R. P. Prajapati *et al.* The above dispersion relation is very lengthy and to study the effect of each parameter we now

reduce the dispersion relation (16) two modes of propagation.

4. ANALYSIS OF THE DISPERSION RELATION

4.1 Longitudinal mode of propagation (K||H)

For this case we assume all the perturbations longitudinal to the direction of the magnetic field i.e. ($k_z = k, k_x = 0$). This is the dispersion relation reduces in the simple form to give

$$\left[- \left\{ \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 V^2}{A} \right)^2 \left(\sigma^2 + \frac{\sigma^2 B v_c}{\sigma + v_c} + \sigma \Omega_v + \Omega_T^2 \right) \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) \right\} - 4 \left(\sigma + \right. \right.$$

$$\left. \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 V^2}{A} \right) \left(\sigma^2 + \frac{\sigma^2 B v_c}{\sigma + v_c} + \sigma \Omega_v + \Omega_T^2 \right) \Omega_x^2 - 4 \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) \left(\sigma^2 + \frac{\sigma^2 B v_c}{\sigma + v_c} + \right.$$

$$\left. \sigma \Omega_v + \Omega_T^2 \right) \Omega_z^2 + 4 \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 V^2}{A} \right) \Omega_x^2 \Omega_T^2 \left. \right] = 0 \quad (17)$$

Equation (17) gives the general dispersion relation for infinite homogeneous, uniformly magnetized, self-gravitating, Rotating viscous, finite electron inertia,

partially ionized plasma, finite electrical Resistivity, thermal conductivity and radiative heat-loss function, when the disturbances are propagating parallel to the

magnetic field. Again for simplicity, the dispersion relation (17) is discussed for axis of rotation is along and perpendicular to the magnetic field separately.

4.1.1. Axis of rotation along the magnetic field

When the axis of rotation is along the magnetic field, we put $\Omega_x = 0$ and $\Omega_z = \Omega$, Then (17) gives

$$-\left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v\right) \left\{ \sigma \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) + \Omega_T^2 \right\} \left[\left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 v^2}{A} \right)^2 + 4\Omega^2 \right] = 0. \quad (18)$$

Equation (18) has three independent factors; each represents the modes of propagation incorporating different parameters.

The first factor of equation(18) is equal to zero.

$$\sigma^2 + \sigma r_1 + \Omega_v v_c = 0. \quad (19)$$

where $r_1 = \{v_c(1 + B) + \Omega_v\}$.

Equation (19) is same as equation (7) earlier obtained by Ali and Bhatia. Equation (19) cannot have a real positive root hence it satisfies the necessary and sufficient condition of stability. The first factor represents stable damped mode due to viscosity of medium, modified by the effect of collision frequency. It is evident from the equation (19) that the condition of stability of the fluid is independent of the magnetic field, electrical resistivity, thermal conductivity, self gravitation, arbitrary radiative heat-loss function and finite electron inertia.

The second factor of equation (18) equating to zero and we obtain following dispersion relation

$$\sigma^4 + \sigma^3(r_1 + \beta) + \sigma^2[\Omega_v v_c + r_1 \beta + \Omega_j^2] + \sigma[v_c\{\Omega_j^2 + \beta \Omega_v\} + \Omega_l^2] + v_c \Omega_l^2 = 0. \quad (20)$$

This dispersion relation for self gravitating fluid incorporated effect of neutral particles, viscosity, thermal conductivity, finite electron inertia and arbitrary radiative heat-loss function. It is evident from equation (20) that the condition of instability is independent of magnetic field, electrical resistivity. The dispersion relation (20) is a fourth degree equation which may be reduced to particular cases so that the effect of each parameter is analyzed separately.

For thermally non-conducting, non-radiating, non viscous, self gravitating fully ionized fluid we have $\alpha = \beta = v_c = \Omega_v = 0$ the dispersion relation (20) reduces to

$$\sigma^2 + \Omega_j^2 = 0. \quad (21)$$

It is clear from equation (21) that when $\Omega_j^2 < 0$, the product of the roots of equation (21) must, therefore, be negative. This implies that at least one root of σ is positive. Hence, the system is unstable. Thus for the cases of equation (21) the condition of instability is

$$\Omega_j^2 = (c^2 k^2 - 4\pi G \rho) < 0. \\ k < k_j = \left(\frac{4\pi G \rho}{c^2} \right)^{1/2}. \quad (22)$$

where k_j is the Jeans wave number. Equation (22) is original Jeans expression for instability. The Jeans length is given as

$$\lambda_j = c \left(\frac{\pi}{G \rho} \right)^{1/2}. \quad (23)$$

The fluid is unstable for all Jeans length $\lambda > \lambda_j$, of Jeans wave number $k < k_j$. It is evident from equation (22) that the Jeans criterion of instability remains unchanged in the presence of neutral particles and viscosity.

For non-radiating but thermally conducting, viscous and self gravitating fluid having neutral particles, the dispersion relation (20) reduces to

$$\sigma^4 + \sigma^3(r_1 + \Omega_k) + \sigma^2[\Omega_v \nu_c + r_1 \Omega_k + \Omega_{j1}^2] + \sigma[\nu_c(\Omega_{j1}^2 + \Omega_v \Omega_k) + \Omega_k \Omega_j^2] + \nu_c \Omega_k \Omega_j^2 = 0. \quad (24)$$

where $c' = \left(\frac{p}{\rho}\right)^{1/2}$ is the isothermal velocity of the sound.

$$\Omega_{j1}^2 = c'^2 k^2 - 4\pi G \rho \quad \text{and} \quad \Omega_k = \frac{\lambda \gamma k^2}{\rho c_p}. \quad (25)$$

It is clear from the constant term of equation (24) that the system leads to instability if $\Omega_{j1}^2 < 0$, which gives

$$c'^2 k^2 - 4\pi G \rho < 0. \quad \text{or} \quad k < k_{j1} = \left(\frac{4\pi G \rho}{c'^2}\right)^{1/2}. \quad (26)$$

Where k_{j1} is the modified Jeans wave number for thermally conducting system and the corresponding modified Jeans length is given as

$$\lambda_{j1} = c' \left(\frac{\pi}{G \rho}\right)^{1/2}. \quad (27)$$

Comparing equation (23) and (27) we see that owing to thermal conduction, the sonic velocity is changed from adiabatic to isothermal one is Jeans expression.

From equation (23) and (27) we obtain

$$\lambda_{j1} = \lambda_j \left(\frac{1}{\gamma}\right)^{1/2}. \quad (28)$$

It is clear from equation (28) that the Jeans length is reduced due to thermal conduction [as $\gamma > 1$], thus the system is destabilized. If we consider self-gravitating and thermally non-conducting plasma incorporated with neutral particles, arbitrary radiative heat-loss function then the condition of instability is given as

$$k < k_{j2} = k_j \left(\frac{\gamma \mathcal{L}_T}{\mathcal{L}_T - \frac{\rho \mathcal{L}_\rho}{T}}\right)^{1/2}. \quad (29)$$

Here k_{j2} is the modified critical wave number due to inclusion of arbitrary radiative heat-loss function. Hence, for wave number $k < k_{j2}$, the system is unstable, therefore, the system begins to break in to fragments of size comparable to $\left(\lambda_{j2} = \frac{2\pi}{k_{j2}}\right)$.

Comparing equation (22) and (29) we find that the critical wave number k_{j2} is very much different from the classical Jeans wave number k_j and depends on derivatives of the arbitrary radiative heat-loss function with respect to local temperature $\mathcal{L}_T$ and local density $\mathcal{L}_\rho$ in the configuration. It is clear from equation (29) that when the arbitrary radiative heat-loss function is independent of density of the configuration (i.e. $\mathcal{L}_\rho = 0$), then $k_{j2} = k_j$ i.e. critical wave number remains unaffected and if the arbitrary radiative heat-loss function is independent of temperature ($\mathcal{L}_T = 0$), then k_{j2} vanishes.

The condition of instability of the system, when combined effect of all the parameters represented by the original dispersion relation (20) is given as

$$\Omega_1^2 = \left[k^2 \left(T\mathcal{L}_T - \rho\mathcal{L}_\rho + \frac{\lambda k^2 T}{\rho} \right) - 4\pi G\rho \left(\frac{T\rho\mathcal{L}_T}{\rho} + \frac{T\lambda k^2}{\rho} \right) \right] < 0. \quad (30)$$

It is evident from condition of instability (31) that the Jeans criterion of instability is modified due to inclusion of thermal conductivity and arbitrary radiative heat-loss function.

From inequality (30) the range of critical wave number k_{j3} is given as

$$k_{j3} = \frac{1}{2^{1/2}} \left[\left\{ \frac{4\pi G\rho}{c^2} + \frac{\rho^2 \mathcal{L}_\rho}{\lambda T} - \frac{\rho \mathcal{L}_T}{\lambda} \right\} \pm \left\{ \left(\frac{4\pi G\rho}{c^2} + \frac{\rho^2 \mathcal{L}_\rho}{\lambda T} - \frac{\rho \mathcal{L}_T}{\lambda} \right)^2 + \frac{16\pi G\rho^2 \mathcal{L}_T}{\lambda c^2} \right\}^{1/2} \right]^{1/2}. \quad (31)$$

If inequalities (31) is applied for a purely temperature dependent arbitrary radiative heat-loss function ($\mathcal{L}_\rho = 0$),

increases with temperature ($\mathcal{L}_T > 0$), then the condition of monotonic instability is given as $k < k_{j1}$. However, if instead the arbitrary radiative heat-loss function decreases with temperature ($\mathcal{L}_T < 0$), the instability arises for k^2 lying between the values $\frac{|\mathcal{L}_T|}{\lambda}$ and $\frac{4\pi G\rho}{c^2}$ for parallel propagation. Furthermore, if it is considered that heat-loss function is purely density dependent ($\mathcal{L}_T = 0$) then the condition of instability is given as

$$k < k_{j4} = \left(\frac{4\pi G\rho}{c^2} + \frac{\rho^2 \mathcal{L}_\rho}{\lambda T} \right)^{1/2}. \quad (32)$$

It is evident from inequality (32) that the critical wave number is increased or decreased, depending on whether the arbitrary radiative heat-loss function is an increasing or decreasing function of the density.

Now equating zero the third factor of equation (18) and after solving we obtain dispersion relation as.

$$\sigma^6 + \sigma^5 A_1 + \sigma^4 A_2 + \sigma^3 A_3 + \sigma^2 A_4 + \sigma A_5 + A_6 = 0. \quad (33)$$

where

$$\begin{aligned} A_1 &= 2 \left[\frac{\Omega_m}{f} + r_1 \right], \\ A_2 &= \left[\frac{\Omega_m^2}{f^2} + r_1^2 + \frac{4\Omega_m}{f} r_1 + 2\Omega_v v_c + \frac{2k^2 v^2}{f} + 4\Omega^2 \right], \\ A_3 &= 2 \left[\frac{k^2 v^2}{f} \left(\frac{\Omega_m}{f} + r_1 + v_c \right) + \frac{\Omega_m}{f} (r_1^2 + 2\Omega_v v_c + 4\Omega^2) + \frac{\Omega_m^2}{f^2} r_1 + r_1 \Omega_v v_c + 4v_c \Omega^2 \right], \\ A_4 &= \left[\frac{v^2 k^2}{f^2} (2r_1 \Omega_m + 2\Omega_v v_c f + 2\Omega_m v_c + 2r_1 v_c f + v^2 k^2) + \frac{\Omega_m^2}{f^2} (r_1^2 + 2\Omega_v v_c + 4\Omega^2) + \frac{\Omega_m v_c}{f} \left(\frac{16\Omega^2 \Omega_m}{f} + 4r_1 \Omega_v \right) + v_c^2 (4\Omega^2 + \Omega_v^2) \right]. \end{aligned}$$

$$A_5 = 2 \left[\frac{V^2 k^2}{f^2} (V^2 k^2 v_c + \Omega_v v_c^2 f + r_1 v_c \Omega_m + \Omega_v \Omega_m v_c) + \frac{v_c^2 \Omega_m}{f} (4\Omega^2 + \Omega_v^2) + \frac{v_c \Omega_m^2}{f^2} (4\Omega^2 + r_1 \Omega_v) \right].$$

$$A_6 = \frac{v_c^2}{f^2} (\Omega_v^2 \Omega_m^2 + 2\Omega_v \Omega_m V^2 k^2 + V^4 k^4 + 4\Omega^2 \Omega_m^2).$$

This dispersion relation (33) involves magnetic field, viscosity, finite electrical resistivity, finite electron inertia, rotation and the effect of the neutral particles, but it does not involve thermal conductivity, arbitrary radiative heat-loss function and self gravitation. Since the coefficient of the equation (33) are all positive including the constant term, therefore, this equation cannot have a positive roots which means that system represented by equation (30) is stable. Thus, this equation gives Alfvén mode modified by the dispersion effect of the neutral particles.

4.1.2. Axis of rotation perpendicular to the magnetic field

In the case of a rotation axis perpendicular to the magnetic field we put $\Omega_x = \Omega$, and $\Omega_z = 0$ in the dispersion relation (17) and this gives.

$$\begin{aligned} & - \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 V^2}{A} \right) \left[\left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 V^2}{A} \right) \left\{ \sigma \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) + \Omega_T^2 \right\} \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) + 4 \left\{ \sigma \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) + \Omega_T^2 \right\} \Omega^2 - 4\Omega_T^2 \Omega^2 \right] = 0. \end{aligned} \quad (34)$$

This dispersion relation is the product of two independent factors. These factors show the mode of propagations

incorporating different parameters as discussed below.

The first factors of equation (34) is obtained

$$\sigma^3 + \sigma^2 \left(\frac{\Omega_m}{f} + r_1 \right) + \sigma \left(\frac{\Omega_m r_1}{f} + \Omega_v v_c + \frac{k^2 v_c^2}{f} \right) + \frac{v_c V^2 k^2}{f} + \frac{\Omega_v \Omega_m v_c}{f} = 0. \quad (35)$$

This equation (35) represents general dispersion relation of the system representing the effect of magnetic field, electron inertia, finite resistivity, viscosity and collision frequency of neutral particles. This third order equation cannot have a real positive roots, this means that the system represented by equation (35) is stable. Equation (35) gives Alfvén mode modified by the effect of resistivity and electron inertia.

The second factor of (34) gives, on substituting the values of Ω_T^2 the following ninth order polynomial equation

$$\sigma^9 + \sigma^8 C_1 + \sigma^7 C_2 + \sigma^6 C_3 + \sigma^5 C_4 + \sigma^4 C_5 + \sigma^3 C_6 + \sigma^2 C_7 + \sigma C_8 + C_9 = 0. \quad (36)$$

The dispersion relation (36) is cumbersome, so its constant term of the last coefficients gives the condition of instability. The condition of instability from the constant term of (36) is given by

$$\nu_c^3 \Omega_\nu \Omega_I^2 \left(\frac{\Omega_\nu \Omega_m}{f} + \frac{V^2 k^2}{f} \right) < 0.$$

The condition of instability is obtained from constant term of equation (36) and gives as

$$\Omega_I^2 = k^2 \alpha - 4\pi G \rho \beta < 0. \quad (37)$$

which is same as equation (30).

4.2. TRANSVERSE PROPAGATION

For this case we assume all the perturbations are propagating perpendicular to the direction of the magnetic field, for, our convenience, we take $k_x = k$, and $k_z = 0$, the general dispersion relation (16) reduces to

$$\begin{aligned} & \left[- \left\{ \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu + \frac{K^2 V^2}{A} \right) \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) \left\{ \sigma \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) + \Omega_T^2 \right\} \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) \right\} - \right. \\ & 4 \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu + \frac{K^2 V^2}{A} \right) \left\{ \sigma \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) + \Omega_T^2 \right\} \Omega_x^2 - 4 \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) \left\{ \sigma \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) + \Omega_T^2 \right\} \Omega_z^2 \\ & \left. + 4 \Omega_z^2 \Omega_T^2 \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) + \frac{K^2 V^2 \Omega_T^2}{A} \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) + \frac{4 K^2 V^2 \Omega_x^2 \Omega_T^2}{A} \right] = 0. \end{aligned} \quad (38)$$

The dispersion relation (38) shows the influence of viscosity, rotation, finite electron inertia and partially-ionized plasmas.

4.2.1. Axis of rotation along magnetic field

When the axis of rotation is along the magnetic field, we put $\Omega_x = 0$ and $\Omega_z = \Omega$, the dispersion relation (38) reduces to

$$\left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) \left[- \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu + \frac{K^2 V^2}{A} \right) \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) \left\{ \sigma \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) + \Omega_T^2 \right\} - 4 \left\{ \sigma \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) + \Omega_T^2 \right\} \Omega^2 + 4 \Omega^2 \Omega_T^2 + \left(\sigma + \frac{\sigma B \nu_c}{\sigma + \nu_c} + \Omega_\nu \right) \Omega_T^2 \frac{V^2 k^2}{A} \right] = 0. \quad (39)$$

Equation (39) has two independent factors, each representing a different mode of propagation. The first factor of (39) gives same result as given in (19).

The second factor of equation (39) gives the nine order equation.

$$\sigma^9 + \alpha_1 \sigma^8 + \sigma^7 \alpha_2 + \sigma^6 \alpha_3 + \sigma^5 \alpha_4 + \sigma^4 \alpha_5 + \sigma^3 \alpha_6 + \sigma^2 \alpha_7 + \sigma \alpha_8 + \alpha_9 = 0. \quad (40)$$

$$\alpha_9 = \left(\frac{\nu_c^2 \Omega_\nu^3 \Omega_m \Omega_I^2}{f} \right).$$

Equation (40) represents the dispersion relation for transverse propagating through finite homogeneous, self-gravitating, viscous magnetized partially ionized plasma having finite electrical resistivity, rotation, radiative effects, electron inertia with the effect of neutral particles.

It can be seen that when $\Omega_I^2 < 0$, The constant term α_9 of the dispersion relation (40) will be negative. This implies that at least one root of is positive, hence the system is unstable. So the condition of

instability for transverse mode of propagation is given as

$$\Omega_I^2 = k^2 \alpha - 4\pi G \rho \beta < 0. \quad (41)$$

which is same as equation (30).

4.2.2 Axis of rotation perpendicular to the magnetic field

We now analyze the wave propagation in transverse direction of external magnetic field considering the rotation of the magnetic field, we put $\Omega_x = \Omega$, and $\Omega_z = 0$ in the dispersion relation (41) and this gives.

$$\left[\left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right)^2 + 4\Omega^2 \right] \left[\frac{v^2 k^2}{A} \Omega_I^2 - \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v + \frac{K^2 v^2}{A} \right) \left\{ \sigma \left(\sigma + \frac{\sigma B v_c}{\sigma + v_c} + \Omega_v \right) + \Omega_I^2 \right\} \right] = 0. \quad (42)$$

where

$$B_1 = \left[\frac{\Omega_m}{f} + 2r_1 + \beta \right].$$

$$B_2 = \left[\frac{\Omega_m}{f} (2r_1 + \beta) + 2r_1 \beta + 2\Omega_v v_c + r_1^2 + \Omega_j^2 \right].$$

$$B_3 = \left[\frac{\Omega_m}{f} (2r_1 \beta + 2\Omega_v v_c + r_1^2 + \Omega_j^2) + \frac{v^2 k^2}{f} (r_1 + v_c + \beta) + \Omega_j^2 r_1 + v_c \Omega_j^2 + \Omega_I^2 + 2\Omega_v v_c \beta + 2r_1 \Omega_v v_c + r_1^2 \beta \right].$$

$$B_4 = \left[\frac{\Omega_m}{f} (\Omega_j^2 r_1 + v_c \Omega_j^2 + \Omega_I^2 + 2\Omega_v v_c \beta + 2r_1 \Omega_v v_c + r_1^2 \beta) + \frac{v^2 k^2}{f} (v_c \beta + r_1 v_c + \Omega_v v_c + r_1 \beta) + 2\beta r_1 \Omega_v v_c + \Omega_v^2 v_c^2 + r_1 \Omega_j^2 v_c + \Omega_I^2 v_c + r_1 \Omega_I^2 + \Omega_v v_c \Omega_j^2 \right].$$

$$B_5 = \left[\frac{\Omega_m}{f} (2\beta r_1 \Omega_v v_c + \Omega_v^2 v_c^2 + r_1 \Omega_j^2 v_c + \Omega_I^2 v_c + r_1 \Omega_I^2 + \Omega_v v_c \Omega_j^2) + \frac{v^2 k^2}{f} (v_c \beta r_1 + \Omega_v v_c \beta + \Omega_v v_c^2) + \Omega_v^2 v_c^2 \beta + r_1 \Omega_I^2 v_c + \Omega_v v_c^2 \Omega_j^2 + \Omega_I^2 \Omega_v v_c \right].$$

$$B_6 = \left[\frac{\Omega_m}{f} (\Omega_v^2 v_c^2 \beta + r_1 \Omega_I^2 v_c + \Omega_I^2 v_c \Omega_v + \Omega_v v_c^2 \Omega_j^2) + \frac{\Omega_v v_c^2 v^2 k^2 \beta}{f} + \Omega_v v_c^2 \Omega_I^2 \right].$$

$$B_7 = \left(\frac{\Omega_v v_c^2 \Omega_m \Omega_I^2}{f} \right).$$

equation (43) seventh degree equation, where the constant term B_7 is given as

$$B_7 = \left(\frac{\Omega_v v_c^2 \Omega_m \Omega_I^2}{f} \right). \quad (46)$$

The first factor of equation (42) gives, on substituting the values of Q_2 and R_1 gives,

$$\sigma^4 + 2r_1 \sigma^3 + (2\Omega_v v_c + r_1^2 + 4\Omega^2) \sigma^2 + (2r_1 \Omega_v v_c + 8\Omega^2 v_c) \sigma + (\Omega_v^2 + 4\Omega^2) v_c^2 = 0. \quad (43)$$

Equation (43) represents damped stable mode depending upon the viscosity of the medium, rotation and collision effects of the neutral particles.

The second factor of equation (42) gives, on substituting the value of A and Ω_I^2 gives

$$\sigma^7 + B_1 \sigma^6 + \sigma^5 B_2 + \sigma^4 B_3 + \sigma^3 B_4 + \sigma^2 B_5 + \sigma B_6 + B_7 = 0. \quad (44)$$

If the condition $\Omega_1^2 < 0$ is satisfied, the above equation admits at least one real positive root which leads to the instability of the system.

CONCLUSION

In the present paper we discussed the problem of gravitational instability of partially ionized rotating viscous plasma including the effect of resistivity, finite electron inertia and radiative heat-loss function. The general dispersion relation for gravitational instability is obtained, which is further reduced for some special cases to see the individual effects of rotation, thermal conductivity, finite resistivity, magnetic field, viscosity, finite electron inertia and radiative heat-loss function. It is found that the viscosity has a damping effect but does not affect the Jeans Criterion of instability. It is also found that the Jeans criterion is unaffected by the collision effects of neutral particles in all the modes of propagation. Thermal conductivity modifies the Jeans criterion of instability in replacing adiabatic sound velocity by isothermal velocity. Jeans criterion of instability is modified by Alfvén velocity in perpendicular direction.

In the transverse mode of propagation with axis of rotation parallel to the magnetic field, we find a gravitating thermal mode influenced by finite electron inertia, rotation, thermal conductivity, collision frequency between two component of plasma and arbitrary radiative heat loss function. The expression of critical jeans wave number both are modified due to the presence of finite electron inertia, magnetic field and rotation and it shows a stabilizing influence.

REFERENCES

1. J. H. Jeans, "The Stability of Spherical Nebula", *Phil. Trans. Roy. Soc. London*, 199 A, 1. (1902).
2. S. Chandrasekhar, "*Hydrodynamics and Hydromagnetic Stability*", (Clarendon Press, Oxford, 1961).
3. A. G. Pacholczyk, and J. S. Stodolkiewicz, "On the Gravitational Instability of Some Magnetohydrodynamical Systems of Astrophysical Interest", *Acta Astron.*, 10, 1, (1960).
4. P. K. Bhatia and R.P.S. Chhonkar: "Larmor radius effects on the instability of a rotating layer of a self-gravitating plasma", *Astrophys. Space Sci.* 115, 327-344 (1985).
5. G. B. Field, "Thermal Instability", *Astrophys. J.* 142, 531 (1965).
6. C. B. Dwiwedi, R. Singh and K. Avinash, "Effect of Radiative Condensation on Jeans Instability", *Physics Scripta*, 53, pp. 760 (1996).
7. M. Aggrawal and S. P. Talwar, "Equilibrium of Rotating Degenerate Fluid Cylinders", *Publi. Astron. Soc. Japan*, 21, pp. 337 (1969).
8. S. Shaikh and A. Khan, "Instability Of Thermally Conducting Self-Gravitating System", *J. Mod. Phys.*, 1, pp 77-82, (2010).
9. R. K. Pensia, V. Ghorela, and R. K. Chhajlani, "Magneto Thermal Instability Of Self-Gravitational Viscous Hall Plasma In The Presence Of Suspended Particle", *Acta Ciencia Indica* 25(2), pp. 141-148, (2009).
10. R. K. Pensia, and V. Ghorela, and R.K. Chhajlani, "Thermal Instability of self-gravitating rotating gaseous plasma with

- generalized Ohm's law", *Ultra Scientist* 20(3), 541, (2008).
11. R.P. Prajapati, A.K. Parihar and R.K. Chhajlani., "Self-gravitational instability of rotating anisotropic heat-conducting plasma", *Phys. Plasmas*, 15, 012107, (2008).
 12. C.Uberio, "Electron-Inertia Effects on the Transverse Gravitational Instability", *J. Plasma Fusion Res.Ser.8* 823(2009).
 13. R.P. Prajapati and R.K. Chhajlani, "Effect of dust temperature on radiative condensation instability of self-gravitating magnetized dusty plasma", *Phys. Scr.*, 81, 045501 (2010).
 14. Yang, C-C., Gruendl, R.A., Chu, Y-H., Mac Low, M.-M., Fukui, Y., "Large-Scale Gravitational Instability and Star Formation in the Large Magellanic Cloud", *Astrophys. J.* 671,374 (2007).
 15. Jacobs, G., Shukla, P.K., "Stability of molecular clouds in partially ionized self-gravitating space plasmas": *J. Plasma Phys.*,71, 487 (2005).
 16. Inutsuka,S., Koyama, H., Inoue, T., "The Role of Thermal Instability in Interstellar Medium": *AIP conf. Proc.*784, 318 (2005).
 17. Menou, K., Balbus, A.S., Spruit, C.H., "Local Axisymmetric Diffusive Stability of Weakly Magnetized, Differentially Rotating, Stratified Fluids": *Astrophys. J.* 607, 564 (2004).
 18. Kim. W. and Narayan, R., "Turbulent Mixing in Clusters of Galaxies": *Astrophys. J.* 596, 889 (2003).
 19. Shadmehri, M., Dib, S., "Magneto-thermal condensation modes including the effects of charged dust particles": *Mon. Not. R. Astron. Soc.*, 395, 985 (2009).